Compact Letter Displays to Concisely Represent Pairwise Sensory Information

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Observation

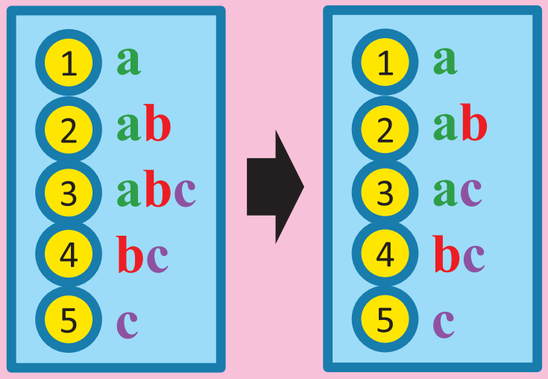
Pairwise comparisons appear in many forms throughout sensory science:
Statistical difference, equivalence, similarity, compatibility, …

Challenging Question

How to best represent pairwise information?

Example: Multiple Statistical Comparisons

Line displays cannot account for unequal variance due to sample size differences, and letter displays often use unnecessary letters



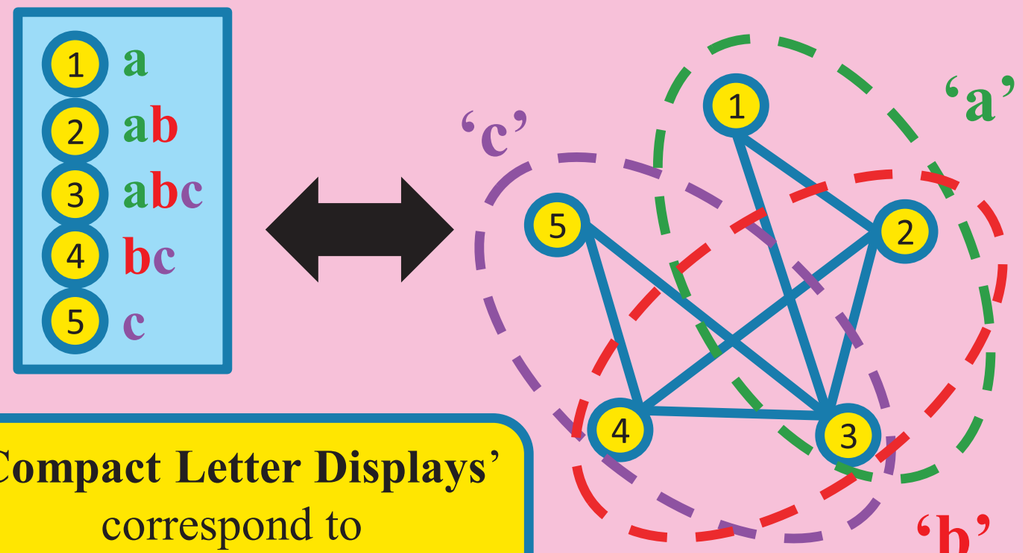
The ‘b’ on 3 carries no new information

Key Insight

Pairwise information ↔ Network graphs

Compact Letter Displays

When a graph is formed from pairwise information, letter displays correspond to ‘**clique coverings**’



‘Compact Letter Displays’ correspond to minimal coverings

As number of items increases, display reduction can become substantial

Product	Letters
1	a
2	ab
3	abc
4	abc
5	abcd
6	bcde
7	bcde
8	bcdef
9	bcdef
10	bcdefg
11	bcdefgh
12	bcdefgh
13	bcdefghi
14	bcdefghi
15	bcdefghi
16	cdefghi
17	defghi
18	efghi
19	fghi
20	ghi
21	hi
22	i

Product	Letters
1	a
2	ab
3	abc
4	abc
5	abcd
6	be
7	be
8	bf
9	bf
10	bfg
11	bfh
12	bfh
13	bi
14	bi
15	bi
16	cefi
17	defi
18	efi
19	fi
20	ghi
21	hi
22	i

In this example, 103 total letters were reduced to 55

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